Finite top mass effects in NNLO Higgs production

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arXiv:0801.2544 [hep-ph] (NP B)
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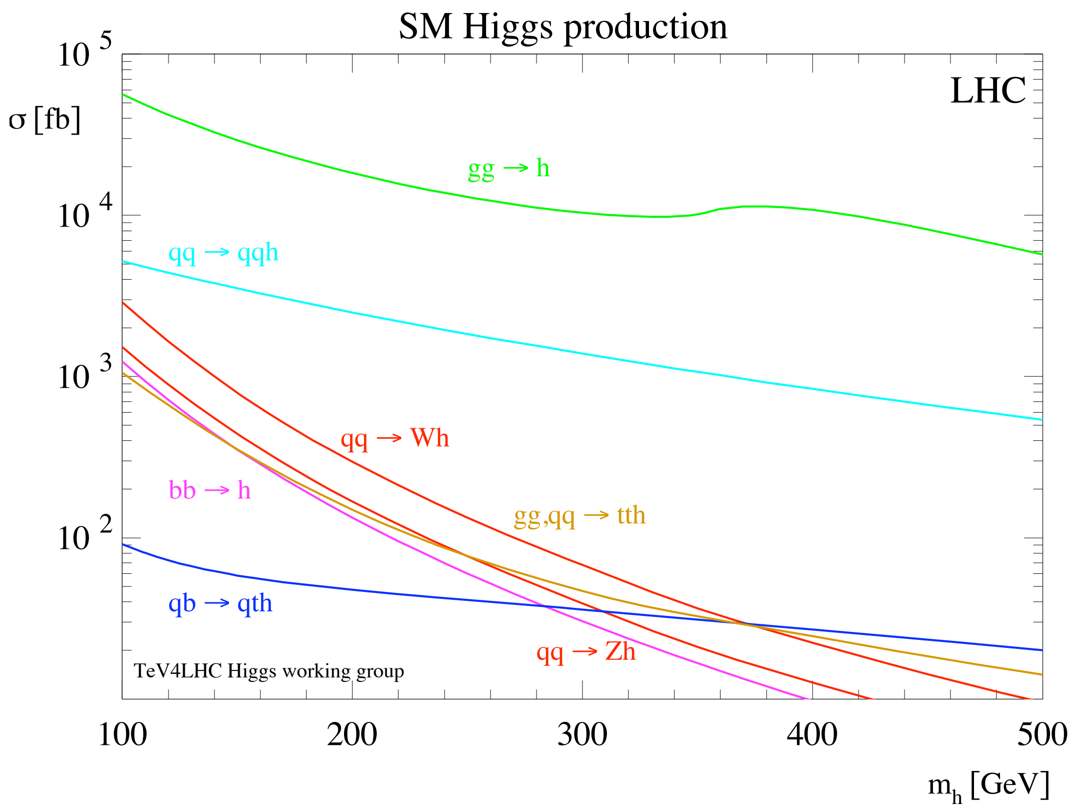
arXiv:0912.2104 [hep-ph] (EJP C)

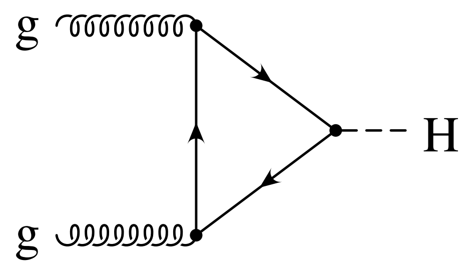
Outline

- Higgs production at the LHC
  - The heavy top approximation
- Top mass effects at NNLO
  - Asymptotic expansion
  - Problems at small-x
- High-energy limit and k_T-factorization
- Matched cross-section
- Conclusions

Higgs production at the LHC

- The Higgs boson is the missing particle of the SM
- Its discovery is the main reason the LHC has been built for





In this talk I will focus on the inclusive cross-section

- The main production channel is gluon-gluon fusion via a quark loop

QCD corrections

- The cross-section can be computed in perturbative QCD

$$\hat{\sigma}_{ij}(x,\tau;\alpha_s) = \sigma_0(\tau)\Big[\delta_{ig}\delta_{jg}\delta(1-x) + \frac{\alpha_s}{\pi}C^{(1)}_{ij}(x,\tau) + \left(\frac{\alpha_s}{\pi}\right)^2 C^{(2)}_{ij}(x,\tau) + \ldots\Big]$$

$$x = \frac{m_H^2}{\hat{s}}\,, \qquad \tau = \frac{4m_t^2}{m_H^2}$$

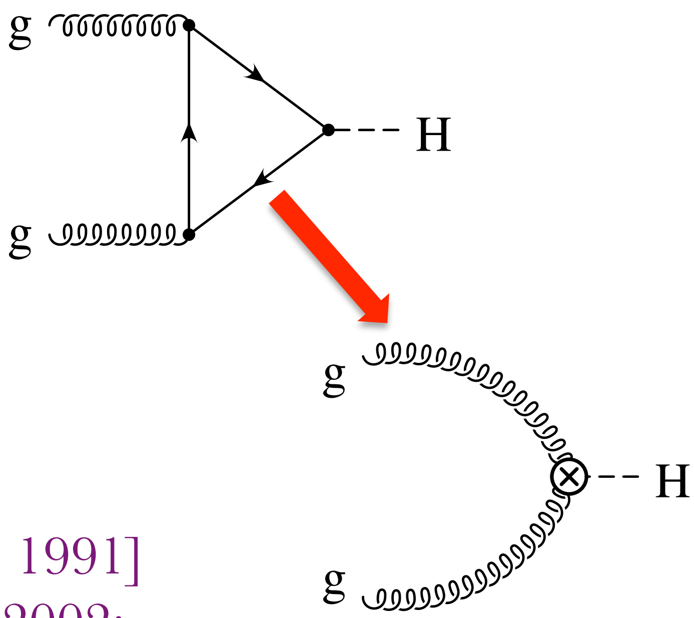
- NLO corrections turn out to be huge (~ 100 %) [Spira et al. 1995]
- The next order is needed to asses the convergence of the series
- The full calculation is beyond the current reach (diagrams with up to 3 loops and massive internal lines)

The heavy top approximation

- EW precision data tell us that the SM Higgs mass should be $\lesssim$ 300 GeV
- This is well below the two top threshold: $\tau \gg 1$
- We can integrate out the top quark and work in an effective theory (EFT)

$$\mathcal{L}_{eff} = -\frac{H}{4v} C_1 G_{\mu\nu} G^{\mu\nu}$$

$$C_1 = -\frac{1}{3}\frac{\alpha_s}{\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \cdots\right\}$$



- Major benefit: one less loop

- NLO [Spira et al. 1991; Dawson, 1991]
- but also NNLO [Anastasiou and Melnikov, 2002; Harlander and Kilgore, 2002; Ravindran, Smith and van Neerven, 2003]

How good is it ?

- The top mass dependence is usually kept at LO, while higher orders are computed in the EFT:

$$\sigma = \sigma^{LO}(m_t) \left(\frac{\sigma}{\sigma^{LO}}\right)_{m_t \to \infty}$$

- When tested against exact NLO the EFT is accurate at the percent level for $m_H < 2\ m_t$

- Surprisingly the agreement is of order 10 % also for $m_H \sim 1$ TeV !

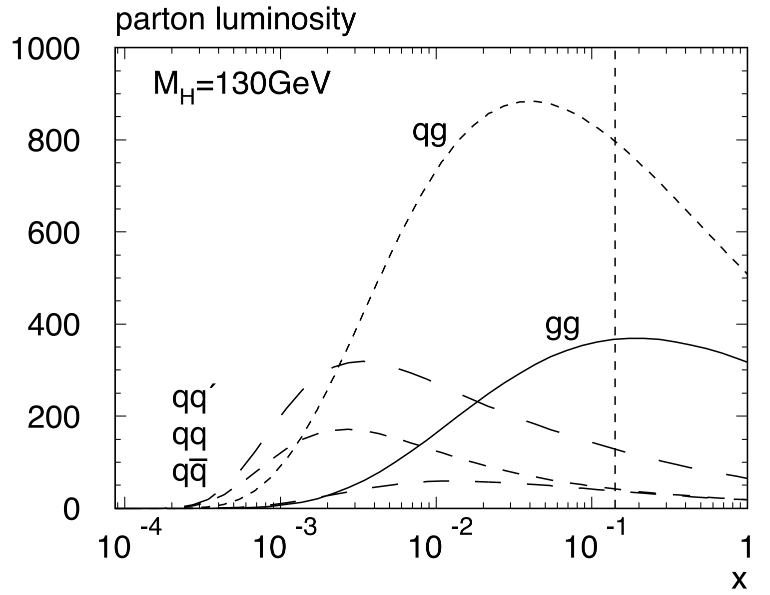
What is the reason for this spectacular agreement ?

Dominant contributions

- The hadronic cross-section is dominated by soft and virtual terms (delta and plus)
- These contributions are almost insensitive to the top mass
- For instance the NLO coefficient function in the gg channel is:

$$
\begin{aligned}
C^{(1)}(x,\tau) &= (\pi^2+\omega(\tau))\delta(1-x) - xP_{gg}(x)\ln x + \mathcal{R}_{gg}(x,\tau) \\
&+ 12\left[\left(\frac{\ln(1-x)}{1-x}\right)_+ - x[2-x(1-x)]\ln(1-x)\right]
\end{aligned}
$$

$$
\begin{aligned}
C^{(1)}(x,\infty) &= (\pi^2+\frac{11}{2})\delta(1-x) - xP_{gg}(x)\ln x - \frac{11}{2}(1-x)^3 \\
&+ 12\left[\left(\frac{\ln(1-x)}{1-x}\right)_+ - x[2-x(1-x)]\ln(1-x)\right]
\end{aligned}
$$



- This should remain true at NNLO as well
- Can we make a more quantitative statement ?
- We can compute top mass suppressed contributions to the NNLO cross-section

Asymptotic expansion

- Full NNLO calculation with top mass not currently feasible
- One can perform an asymptotic expansions of the Feynman diagrams

  [e.g. Smirnov 2002]
- The cross-section can be written as

$$\sigma = \sum_n \left(\frac{m_H^2}{4m_t^2} \right)^n \sigma_n$$

- The first term is the EFT one

- Top mass suppressed corrections to NLO known for a long time

  [Dawson, Kauffman 1993]
- Now also computed at NNLO by two different groups

  [Harlander, Ozeren 2009
  Pak, Rogal, Steinhauser 2009]
- Tools exist to automatize the calculation (not going into the details)

… however …

Problems at large $\hat{s}$

- The asymptotic expansion assumes

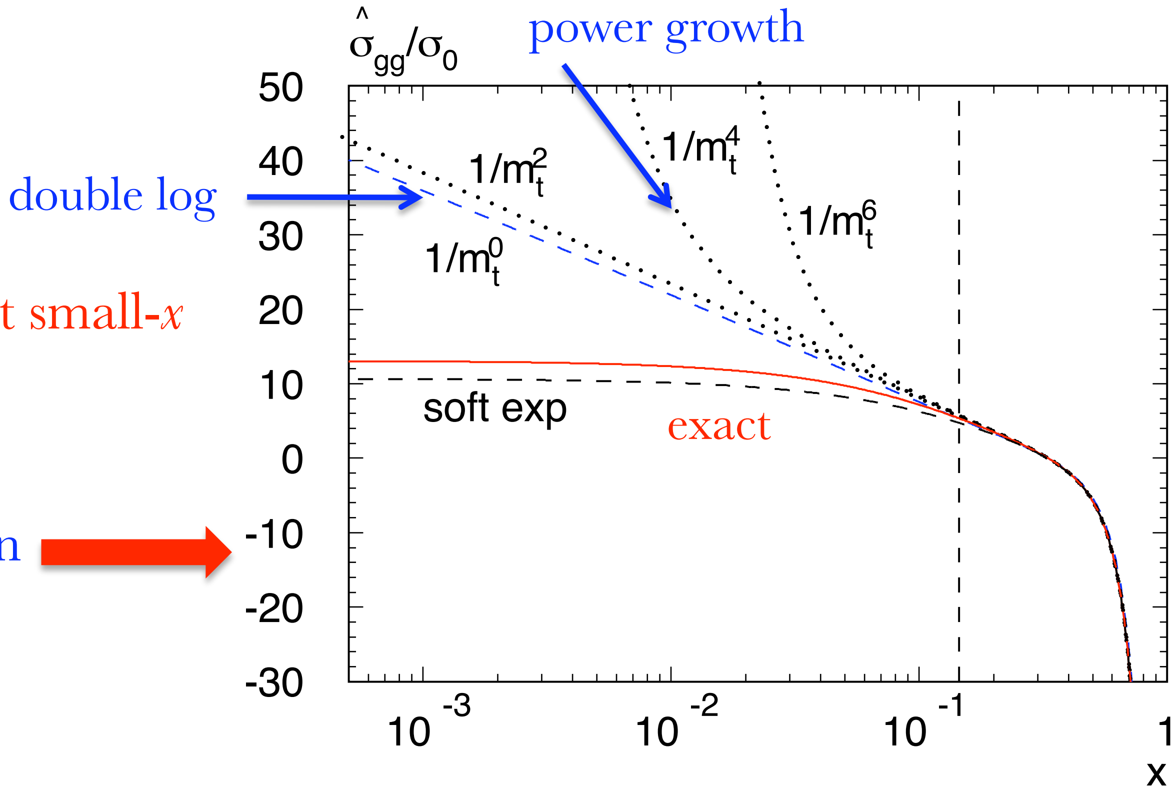
$$\sqrt{\hat{s}}, m_H \ll 2m_t$$

- Clearly at the LHC the partonic c.o.m. energy can reach values far beyond m_t
- The expansion breaks down in the high-energy region
- This breakdown manifests itself in inverse powers of

$$\frac{\hat{s}}{m_t^2} = \frac{m_H^2}{m_t^2}\frac{1}{x}$$

- Spurious power-like growth at small-x appears disastrous !

NLO coefficient function



So far…

- In order to compute finite top mass corrections at NNLO we can use asymptotic expansion
- This is OK in the region below threshold, where the top mass is the largest scale in the process
- This region dominates the cross-section after convolution with parton luminosity
- We need a different method to compute the hard tail of the partonic coefficient functions at NNLO

So far…

- In order to compute finite top mass corrections at NNLO we can use asymptotic expansion
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- We can use k_T-factorization

QCD factorizations

- Hard processes : collinear factorization $Q^2 \gg \Lambda^2_{QCD}$

$$\Sigma(\tau_h, Q^2) = \int_{\tau_h}^{1} \frac{dx_1}{x_1} \int_{\tau_h}^{1} \frac{dx_2}{x_2} \hat{\Sigma}_{gg}\left(\frac{\tau_h}{x_1 x_2}, \frac{Q^2}{\mu^2}\right) F(x_1, \mu^2) F(x_2, \mu^2)$$

longitudinal momentum fractions of the
on-shell incoming partons

parton densities

QCD factorizations

- Hard processes : collinear factorization $\quad Q^2 \gg \Lambda^2_{QCD}$

$$\Sigma(\tau_h, Q^2) = \int_{\tau_h}^1 \frac{dx_1}{x_1} \int_{\tau_h}^1 \frac{dx_2}{x_2} \hat{\Sigma}_{gg}\left(\frac{\tau_h}{x_1 x_2}, \frac{Q^2}{\mu^2}\right) F(x_1, \mu^2) F(x_2, \mu^2)$$

- High energy processes: k_T-factorization $\quad S \gg Q^2 \gg \Lambda^2_{QCD}$

$$\Sigma(\tau_h, Q^2) \quad = \quad \int_{\tau_h}^1 \frac{dx_1}{x_1} \int_{\tau_h}^1 \frac{dx_2}{x_2} \int \frac{d^2 k_{T1}}{\pi k_{T1}^2} \int \frac{d^2 k_{T2}}{\pi k_{T2}^2} \hat{\Sigma}_{gg}^{\text{off}}\left(\frac{\tau_h}{x_1 x_2}, \frac{k_{T1}}{Q}, \frac{k_{T2}}{Q}\right)$$
$$\mathcal{F}(x_1, k_{T1}^2, \mu^2) \mathcal{F}(x_2, k_{T2}^2, \mu^2)$$

transverse momenta of the off-shell incoming partons

unintegrated parton densities

High-energy factorization

- We consider Mellin moments of the off-shell cross section:

$$\sigma(N, M_1, M_2) = \int_0^1 x^{N-1} \int_0^\infty (k_1^2)^{M_1-1} \int_0^\infty (k_2^2)^{M_2-1} \sigma(x, k_1^2, k_2^2)$$

- So that the formula factorizes

$$\begin{aligned}\sigma(N, M_1, M_2) &= \mathcal{H}(N, M_1, M_2)\mathcal{F}(N, M_1)\mathcal{F}(N, M_2) \\ &= \mathcal{H}(N, M_1, M_2) M_1 F(N, M_1) M_2 F(N, M_2)\end{aligned}$$

- To make contact with usual collinear factorization we have introduced the Mellin moments of the integrated PDFs

QCD evolution equations

DGLAP: Q^2 evolution for N moments of the parton density

$$\frac{d}{d\ln(Q^2/\mu^2)} F(N,Q^2) = \gamma(N,\alpha_s) F(N,Q^2)$$

BFKL: small-x evolution for M moments of the parton density

$$\frac{d}{d\ln(1/x)} F(x,M) = \chi(M,\alpha_s) F(x,M)$$

Mellin moments:

logs $\leftrightarrow$ poles

$$\ln^k \frac{Q^2}{\mu^2} \leftrightarrow \frac{1}{M^{k+1}}$$

$$\ln^k \frac{1}{x} \leftrightarrow \frac{1}{N^{k+1}}$$

Duality relations

- At high energy and large Q^2 both BFKL and DGLAP are valid

- They admit the same leading twist solution

$$F(N,M) = \frac{F_0(N)}{M - \gamma(\alpha_s, N)} \quad = \quad \frac{\bar{F}_0(M)}{N - \chi(\alpha_s, M)}$$

DGLAP and BFKL

- The kernels satisfy (consistency) duality relations

$$\begin{aligned} \chi(\gamma(N,\alpha_s),\alpha_s) &= N \\ \gamma(\chi(M,\alpha_s),\alpha_s) &= M \end{aligned}$$

Coefficient functions at high energy

- We define the impact factor in the following way:

$$h(M_1, M_2) = M_1 M_2 \int_0^\infty (k_1^2)^{M_1-1} \int_0^\infty (k_2^2)^{M_2-1} \hat{\sigma}^{\mathrm{off}}$$

- the explicit N dependence is sub-leading, hence we set N=0

- the high energy behaviour is found by inverting the M-Mellin transforms using the pole condition from the evolution equations:

$$\oint \frac{dM_1}{2\pi i} \oint \frac{dM_2}{2\pi i} \left(\frac{Q^2}{\mu^2}\right)^{M_1+M_2} h(M_1, M_2) \frac{F(N)}{M_1 - \gamma_s} \frac{F(N)}{M_2 - \gamma_s}$$

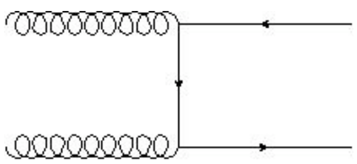
- one obtains:

$$h(\gamma_s(N), \gamma_s(N)) \qquad \text{with} \qquad \gamma_s = \sum_k a_k \left(\frac{\alpha_s}{N}\right)^k$$

LO BFKL anomalous dimension

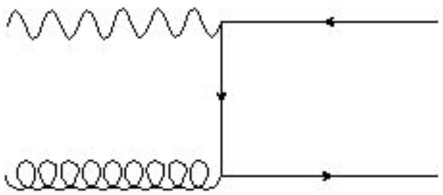
What has been computed so far

- Originally used for heavy flavour production



Catani, Ciafaloni, Hautmann Nucl.Phys.B366:135-188,1991.
Ball, Ellis JHEP 0105:053,2001.

- DIS and DY are more delicate because collinear singularities (due to massless quarks) must be consistently factorized



Catani, Hautmann Nucl.Phys.B427:475-524,1994.
SM, Ball Nucl.Phys.B814:246-264,2009

- Direct photon: final state singularities

Diana, Nucl.Phys.B824:154-167,2010.

Computation in k_t-factorization

- We compute the LO off-shell cross section for

$$g^*(\xi_1) \quad g^*(\xi_2) \to H$$

- The impact factor is

$$h(N, M_1, M_2) \sim M_1 M_2 \int_0^{+\infty} d\xi_1 \xi_1^{M_1 - 1} \int_0^{+\infty} d\xi_2 \xi_2^{M_2 - 1} \frac{\mathcal{A}(\xi_1, \xi_2)}{(1 + \xi_1 + \xi_2)^N}$$

- The form factor ensures that the Mellin integrals have finite radius of convergence when N = 0

$$h(0, M_1, M_2) \sim \sigma_0 m_H^2 \left[1 + s_1(M_1 + M_2) + s_2(M_1^2 + M_2^2) + s_{1,1} M_1 M_2 \ldots\right]$$

- Only single poles (ie single logs) when we identify

$$M_1 = M_2 = \gamma_s\left(\frac{\alpha_s}{N}\right) = \frac{\alpha_s}{\pi}\frac{C_A}{N} + \ldots$$

Partonic results

- We numerically evaluate the coefficient of the leading logarithm at small-*x* in the gg channel
- We then compute the small-*x* behaviour of the other channels using colour charge relations

$$\gamma_{gq} = \frac{C_F}{C_A}\gamma_s + \mathcal{O}\left(\frac{\alpha_s^2}{N}\right)$$

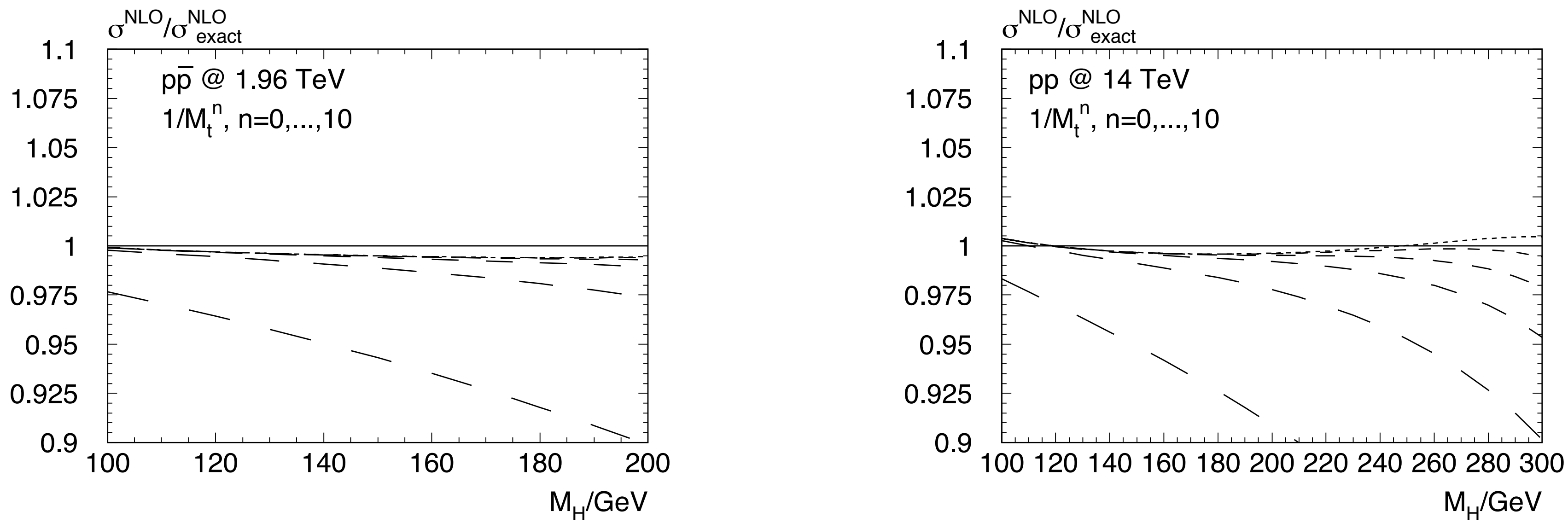
- We obtain

$$\begin{aligned}
C_{gg}(x,\tau) &= \delta(1-x) + \frac{\alpha_s}{\pi}\left[B_{gg}^{(1)}(\tau) + \mathcal{O}(x)\right] \\
&\quad + \left(\frac{\alpha_s}{\pi}\right)^2\left[A_{gg}^{(2)}(\tau)\ln\frac{1}{x} + \mathcal{O}(x^0)\right] + \ldots \\
C_{qg}(x,\tau) &= \frac{\alpha_s}{\pi}\left[B_{qg}^{(1)}(\tau) + \mathcal{O}(x)\right] \\
&\quad + \left(\frac{\alpha_s}{\pi}\right)^2\left[A_{qg}^{(2)}(\tau)\ln\frac{1}{x} + \mathcal{O}(x^0)\right] + \ldots \\
C_{q_i q_j}(x,\tau) &= \frac{\alpha_s}{\pi}\left[\,\mathcal{O}(x)\,\right] \\
&\quad + \left(\frac{\alpha_s}{\pi}\right)^2\left[A_{qq}^{(2)}(\tau)\ln\frac{1}{x} + \mathcal{O}(x^0)\right] + \ldots
\end{aligned}$$

- We checked the NLO coefficients against the full result.

Hadronic results (NLO)

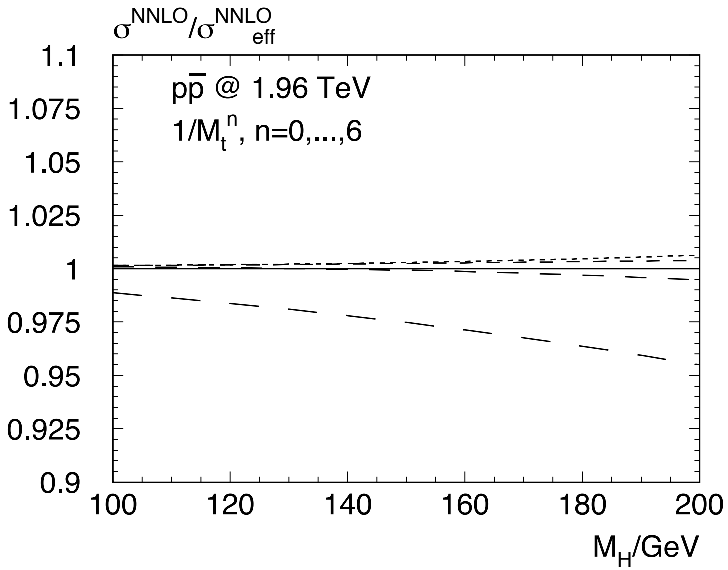
- We construct an approximation to the exact cross-section by matching the $1/m_t$ expansion to the small-*x* limit (with the full m_t dependence)
- In order to test this procedure we first study the NLO case

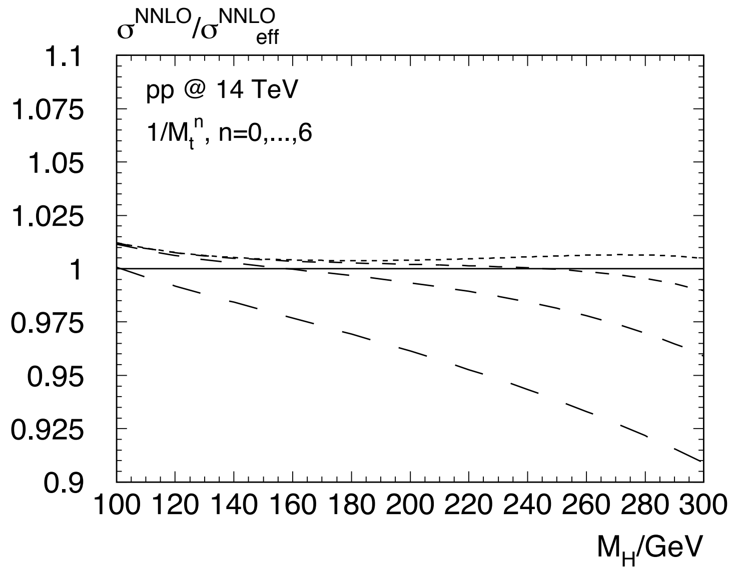


- The convergence of the approximate result toward the exact one is excellent
- We apply the same procedure to the next order

Hadronic results (NNLO)

- At NNLO we compute ratios of the our approximation to the EFT results





- Finite top mass effects at NNLO are below 1% both at the Tevatron and LHC
- The EFT approach is fully justified to NNLO (for the inclusive cross-section)

Conclusions

- I have presented a calculation for Higgs production in g-g fusion to NNLO
- Below threshold finite top mass corrections are included performing asymptotic expansions of the Feynman diagrams
- In the high partonic centre of mass region this approach fails
- The small-*x* limit has been computed using k_T-factorization and then matched to the $1/m_t$ expansion
- Finite top mass effects at NNLO are below 1% both at the Tevatron and LHC
- The EFT approach is fully justified to NNLO (for the inclusive cross-section)

- This calculation is an example of a fruitful interplay between fixed-order and resummation techniques !

Thank you !